Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE 2008

MARKING SCHEME

APPLIED MATHEMATICS

ORDINARY LEVEL

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General Guidelines

1. Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2), 10 (att 3).

2. The marking scheme shows one correct solution to each question. In many cases there are other equally valid methods.

1. Four points *a*, *b*, *c* and *d* lie on a straight level road.
A car, travelling with uniform retardation, passes point *a* with a speed of 30 m/s and passes point *b* with a speed of 20 m/s.
The distance from *a* to *b* is 100 m. The car comes to rest at *d*.

Find **(i)** the uniform retardation of the car

(ii) the time taken to travel from *a* to *b*

(iii) the distance from *b* to *d*

(iv) the speed of the car at *c*, where *c* is the midpoint of [*bd*].

(i) $v^2 = u^2 + 2as$

$20^2 = 30^2 + 2(a)(100)$ 10

$-500 = 200\,a$

$a = -2.5 \text{ m/s}^2$

$\Rightarrow$ retardation $= 2.5 \text{ m/s}^2$ 10

(ii) $v = u + at$

$20 = 30 - 2.5\,t$

$t = 4$ s. 10

(iii) $v^2 = u^2 + 2as$

$0^2 = 20^2 + 2(-2.5)(s)$

$s = 80 \text{ m}$ 10

(iv) $v^2 = u^2 + 2as$

$= 20^2 + 2(-2.5)(40)$

$= 200$

$v = 10\sqrt{2}$ or 14.1 m/s 10

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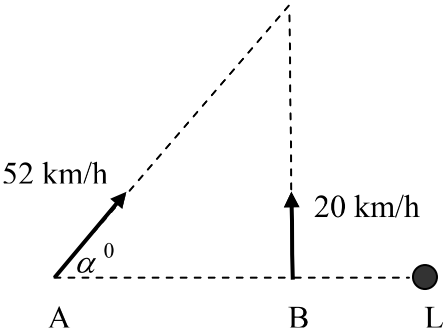
2. Ship A is 432 km due west of ship B.
Ship B is 135 km due west of lighthouse L.
A is travelling at a constant speed of 52 km/h in the direction east α^0 north, where $\tan\alpha = \frac{5}{12}$.

B is travelling due north at a constant speed of 20 km/h.



Find **(i)** the velocity of A in terms of $\vec{i}$ and $\vec{j}$

(ii) the velocity of B in terms of $\vec{i}$ and $\vec{j}$

(iii) the velocity of A relative to B in terms of $\vec{i}$ and $\vec{j}$.

Ship A intercepts ship B after *t* hours.

(iv) Find the value of *t*.

(v) Find the distance from lighthouse L to the meeting point.

(i)	$\vec{V}_A = 52\cos\alpha\,\vec{i} + 52\sin\alpha\,\vec{j}$	5	
	$= 48\,\vec{i} + 20\,\vec{j}$	5	
(ii)	$\vec{V}_B = 0\,\vec{i} + 20\,\vec{j}$	10	
(iii)	$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$	5	
	$= (48\,\vec{i} + 20\,\vec{j}) - (0\,\vec{i} + 20\,\vec{j})$		
	$= 48\,\vec{i} + 0\,\vec{j}$	5	
(iv)	$\text{time} = \frac{432}{48} = 9$ hours	10	
(v)	In 9 hours : B travels 20×9		
	$= 180$ km	5	
	distance $= \sqrt{180^2 + 135^2}$		
	$= 225$ km	5	50

3. A particle is projected from a point on horizontal ground with an initial speed of 25 m/s at an angle β^0 to the horizontal where tan $\beta = \frac{4}{3}$.

(i) Find the initial velocity of the particle in terms of $\vec{i}$ and $\vec{j}$.

(ii) Calculate the time taken to reach the maximum height.

(iii) Calculate the maximum height of the particle above ground level.

(iv) Find the range.

(v) Find the speed and direction of the particle after 3 seconds of motion.

(i)

$\vec{V} = 25\cos\beta\,\vec{i} + 25\sin\beta\,\vec{j}$ 5

$= 15\,\vec{i} + 20\,\vec{j}$ 5

(ii)

$v = u + at$

$0 = 20 + (-10)(t)$

$t = 2 \text{ s}$ 10

(iii)

$s = ut + \tfrac{1}{2}at^2$ or $v^2 = u^2 + 2as$

$= 20(2) - 5(2)^2$ $\quad 0^2 = 20^2 + 2(-10)(s)$

$s = 20 \text{ m}$ $\quad s = 20 \text{ m}$ 10

(iv)

time $= 4$ s

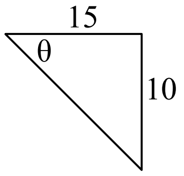
range $= 15(4)$

$= 60$ m 10

(v)

$\vec{V} = (15)\,\vec{i} + (20 + (-10)t)\,\vec{j}$

$\vec{V} = 15\,\vec{i} - 10\,\vec{j}$



speed $= \sqrt{(15)^2 + (-10)^2}$

$= 18.0 \text{ m/s}$ 5

$\tan\theta = \frac{10}{15}$

$\theta = 33.69°$ 5

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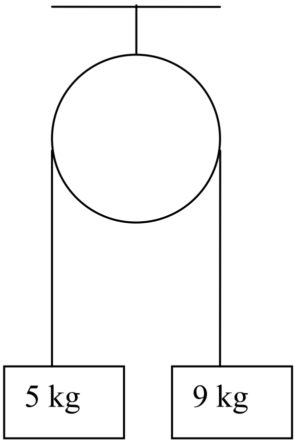
4. **(a)** Two particles of masses 9 kg and 5 kg are connected by a taut, light, inextensible string which passes over a smooth light pulley.

The system is released from rest.

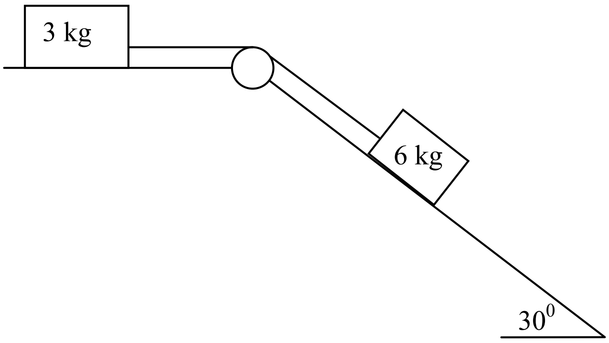
Find **(i)** the common acceleration of the particles

(ii) the tension in the string.



(b) Masses of 3 kg and 6 kg are connected by a taut, light, inextensible string which passes over a smooth light pulley as shown in the diagram.

The 3 kg mass lies on a rough horizontal plane and the coefficient of friction between the 3 kg mass and the plane is μ.



The 6 kg mass lies on a smooth plane which is inclined at 30^0 to the horizontal.

When the system is released from rest each mass travels 1 metre in $\sqrt{2}$ seconds.

Find **(i)** the common acceleration of the masses

(ii) the tension in the string

(iii) the value of μ.

4 (a) (i)

$$T-5g=5a$$ 5

$$9g-T=9a$$ 5

$$a=\frac{40}{14} \text{ or } 2.86\,\text{m/s}^2$$ 5

(ii)

$$T-5g=5a$$

$$T-50=14.29$$

$$T=64.29\ \text{N}$$ 5

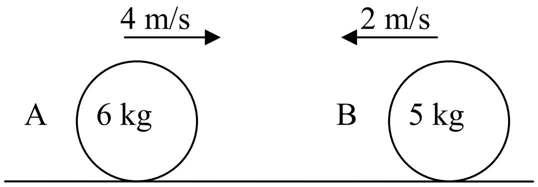
20

4(b)

(i)	$s = ut + \tfrac{1}{2}at^2$		
	$1 = 0 + \tfrac{1}{2}a(2)$	5	
	$a = 1\ \text{m/s}^2$	5	
(ii)	$6g\sin 30 - T = 6a$	5	
	$30 - T = 6$		
	$\text{T} = 24\ \text{N}$	5	
(iii)	$T - \mu R = 3a$	5	
	$24 - \mu(3g) = 3(1)$		
	$30\mu = 21$		
	$\mu = \frac{7}{10}$	5	30

5. A smooth sphere A, of mass 6 kg, collides directly with another smooth sphere B, of mass 5 kg, on a smooth horizontal table.

A and B are moving in opposite directions with speeds of 4 m/s and 2 m/s respectively.



The coefficient of restitution for the collision is $\frac{1}{10}$.

Find **(i)** the speed of A and the speed of B after the collision
(ii) the loss in kinetic energy due to the collision
(iii) the magnitude of the impulse imparted to A due to the collision.

(i)	PCM	$6(4) + 5(-2) = 6v_1 + 5(v_2)$	10	
		$14 = 6v_1 + 5v_2$		
	NEL	$v_1 - v_2 = -e(u_1 - u_2)$		
		$= -\frac{1}{10}(4+2)$	10	
		$= -\frac{6}{10}$		
		$v_1 = 1$ m/s and $v_2 = 1.6$ m/s	10	
(ii)	KE before collision	$= \frac{1}{2}(6)(4)^2 + \frac{1}{2}(5)(-2)^2$		
		$= 58$	5	
	KE after collision	$= \frac{1}{2}(6)(1)^2 + \frac{1}{2}(5)(1.6)^2$		
		$= 9.4$	5	
	KE lost	$= 58 - 9.4$		
		$= 48.6$ J	5	
(iii)	Impulse	$= \lvert (6)(4) - (6)(1) \rvert$		
		$= 18$ Ns	5	50

6. **(a)** Particles of weight 5 N, 2 N, 3 N and 8 N are placed at the points $(p,q), (7,p), (-2,q)$ and $(1,-6)$, respectively.
The co-ordinates of the centre of gravity of the system are $(2,0)$.

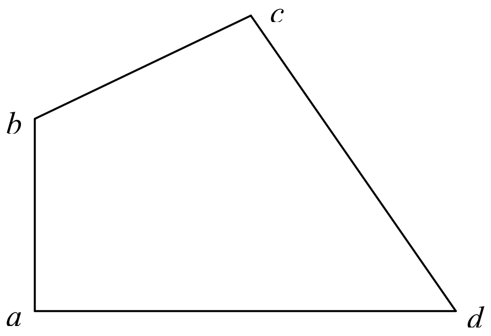
Find **(i)** the value of p

(ii) the value of q.

(b) A quadrilateral lamina has vertices a, b, c and d.

The co-ordinates of the vertices are $a(0,0), b(0,6), c(6,9)$ and $d(12,0)$.

Find the co-ordinates of the centre of gravity of the lamina.



(a)

$$2 = \frac{5(p)+2(7)+3(-2)+8(1)}{18} \qquad 10$$

$$p = 4 \qquad 5$$

$$0 = \frac{5(q)+2(4)+3(q)+8(-6)}{18} \qquad 10$$

$$q = 5 \qquad 5$$

(b)

	area :	c.g.	
abc	$\frac{1}{2}(6)(6)=18$	$(2,5)$	5
acd	$\frac{1}{2}(12)(9)=54$	$(6,3)$	5
lamina	$=72$	(x,y)	

$$(72)(x) = 54(6)+18(2)$$

$$x = 5 \qquad 5$$

$$(72)(y) = 54(3)+18(5)$$

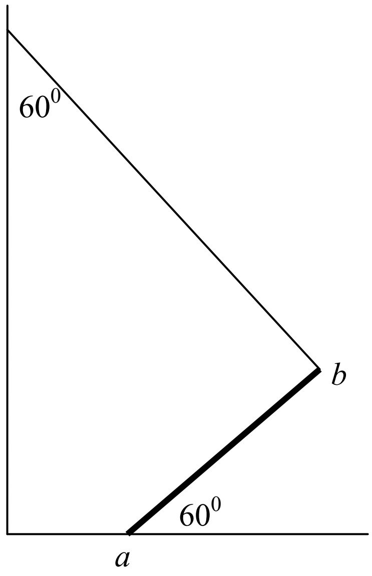
$$y = 3.5 \qquad 5$$

co - ords of c.g. $(5, 3.5)$

$\boxed{50}$

7. A uniform rod, [*ab*], of length 4 m and weight 100 N is smoothly hinged at end *a* to a horizontal floor.
One end of a light inelastic string is attached to *b* and the other end of the string is attached to a vertical wall.

The string makes an angle of 60^0 with the wall and the rod makes an angle of 60^0 with the floor, as shown in the diagram.
The rod is in equilibrium.



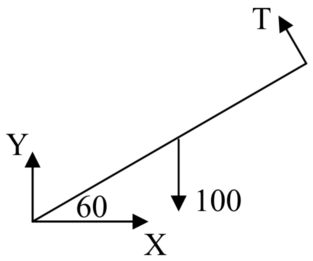
(i) Show on a diagram all the forces acting on the rod [*ab*].

(ii) Write down the two equations that arise from resolving the forces horizontally and vertically.

(iii) Write down the equation that arises from taking moments about point *a*.

(iv) Find the tension in the string.

(v) Find the magnitude of the reaction at the hinge.

(i)  	10
(ii) horiz $T\sin 60 = X$	5
vert $Y + T\cos 60 = 100$	5
(iii) Take moments about *a* : $T(4) = 100(2\cos 60)$	10
(iv) $T = 25$ N	10
(v) $Y + 25\cos 60 = 100$ $\Rightarrow Y = 87.5$ $X = 25\sin 60 = 21.65$ Reaction at *a* $= \sqrt{21.65^2 + 87.5^2}$ $= 90.1$ N	10

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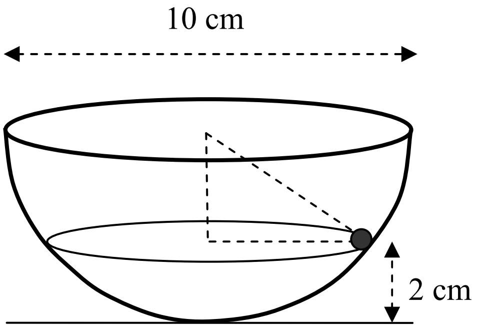
8. **(a)** A particle describes a horizontal circle of radius 2 metres with constant angular velocity ω radians per second.
Its speed is 5 m/s and its mass is 3 kg.

Find **(i)** the value of ω
(ii) the centripetal force on the particle.

(b) A hemispherical bowl of diameter 10 cm is fixed to a horizontal surface.



A smooth particle of mass 2 kg describes a horizontal circle of radius r cm on the smooth inside surface of the bowl.

The plane of the circular motion is 2 cm above the horizontal surface.

(i) Find the value of r.
(ii) Show on a diagram all the forces acting on the particle.
(iii) Find the reaction force between the particle and the surface of the bowl.
(iv) Calculate the angular velocity of the particle.

(a)

(i) $r\omega = v$

$2\omega = 5$

$\Rightarrow \quad \omega = 2.5$ rad/s — 10

(ii) Force $= mr\omega^2$

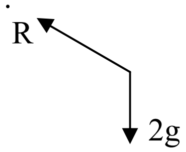
$= (3)(2)(2.5^2)$

$= 37.5$ N — 10

(b)

(i) $r = \sqrt{5^2 - 3^2} = 4$ — 5

(ii) :



5

(iii) $R\sin\alpha = 2g$

$R\left(\frac{3}{5}\right) = 20 \Rightarrow R = \frac{100}{3}$ — 10

(iv) $R\cos\alpha = mr\omega^2$

$\left(\frac{100}{3}\right)\left(\frac{4}{5}\right) = 2(4)\omega^2 \Rightarrow \omega = \sqrt{\frac{10}{3}}$ — 10

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9. **(a)** State the Principle of Archimedes.

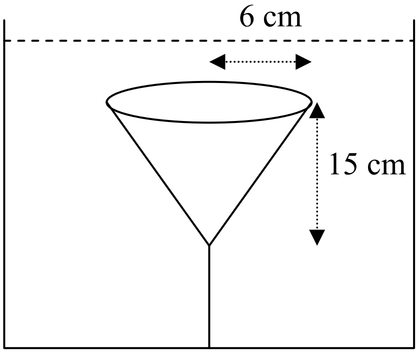
A solid piece of metal has a weight of 28 N.
When it is completely immersed in water the metal weighs 18 N.

Find **(i)** the volume of the metal
(ii) the relative density of the metal.

(b) A right circular solid cone has a base of radius 6 cm and a height of 15 cm.

The relative density of the cone is 0.6 and it is completely immersed in a tank of liquid of relative density 0.9.

The cone is held at rest by a light inextensible vertical string which is attached to the base of the tank.
The upper surface of the cone is horizontal.
Find the tension in the string.



[Density of water = 1000 kg/m^3]

(a)

Principle of Archimedes 5

(i) B = weight of water displaced

$10 = \rho V g = 1000V(10)$ 5

$\Rightarrow V = \dfrac{1}{1000}$ 5

(ii) $\rho = \dfrac{M}{V} = \dfrac{2.8}{0.001} = 2800$ 5

$\Rightarrow$ relative density = 2.8 5

(b)

$\text{B} = T + W$ 5

$\dfrac{W(0.9)}{0.6} = T + W$

$T = \tfrac{1}{2}W$ 5

$T = \tfrac{1}{2}\rho V g$ 5

$= \dfrac{1}{2}\left\{600\left(\tfrac{1}{3}\pi(0.06)^2(0.15)\right)10\right\}$ 5

$T = 1.7 \text{ N}$ 5

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